Edexcel IAL Physics A-level

Topic 4.4: Electric and Magnetic Fields

Notes

4.4 - Electric and Magnetic fields

4.4.92 - Electric fields

A **force field** is an area in which an object experiences a **non-contact force**. Force fields can be represented as **vectors**, which describe the direction of the force that would be exerted on the object, from this knowledge you can deduce the direction of the field. They can also be represented as diagrams containing **field lines**, the distance between field lines represents the strength of the force exerted by the field in that region.

An **electric field** is a **force field** in which charged particles experience a force.

4.4.93 - Electric field strength

Electric field strength (E) is the force per unit charge experienced by an object in an electric field.

$$E = \frac{F}{Q}$$

Where **F** is the force exerted on the object and **Q** is the charge of the object.

This value is **constant** in a uniform field, but **varies** in a radial field. *(Uniform and radial fields are explained in 4.4.115).*

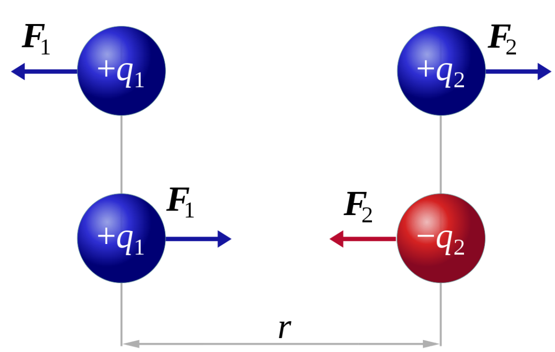
4.4.94 - Coulomb's law

Coulomb's law states that the magnitude of the force between two point charges is **directly proportional to the product of their charges**, and **inversely proportional to the square of the distance** between them.

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

Where $\boldsymbol{\varepsilon_0}$ is the permittivity of free space, $\mathbf{Q_1/Q_2}$ are charges, and **r** is the distance between the charges.

If charges have the **same** sign the force will be **repulsive**, and if the charges have **different** signs the force will be **attractive**.



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4.4.95 - Electric field strength in a radial field

Point charges form a radial electric field, you can calculate the **electric field strength** in such a field by using the following formula:

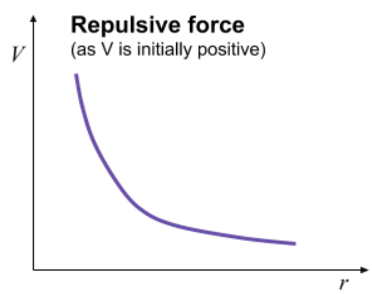
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

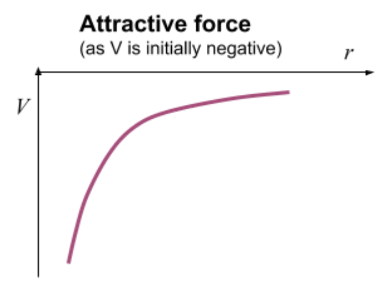
Where ε_0 is the permittivity of free space, **Q** is the charge, and **r** is the distance from the centre of the charge to the point of interest.

4.4.96 - Electric potential

Absolute electric potential (V) at a point is the potential energy per unit charge of a positive point charge at that point in the field. The absolute magnitude of electric potential is **greatest at the surface of a charge**, and as the distance from the charge increases, the potential decreases, so **electric potential at infinity is zero**.

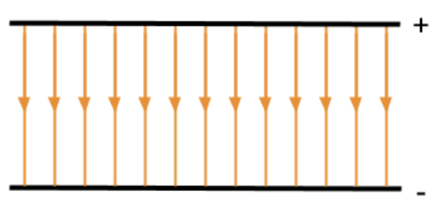
Whether the value of potential is negative or positive depends on the sign of the charge (Q), when the charge is positive, **potential is positive and the charge is repulsive**, when the charge is negative, **potential is negative and the force is attractive**.





4.4.97 - Electric fields between parallel plates

You can form a **uniform** electric field using a pair of parallel plates with a potential difference across them as shown below.



Uniform field

You can calculate the **electric field strength (E)** in an electric field formed between parallel plates by dividing the potential difference across the plates by the distance between them:

$$E = \frac{V}{d}$$

Where **V** is the potential difference across the parallel plates, and **d** is the distance between them.

4.4.98 - Electric potential in a radial field

To find the value of electric potential in a **radial field** you can use the formula:

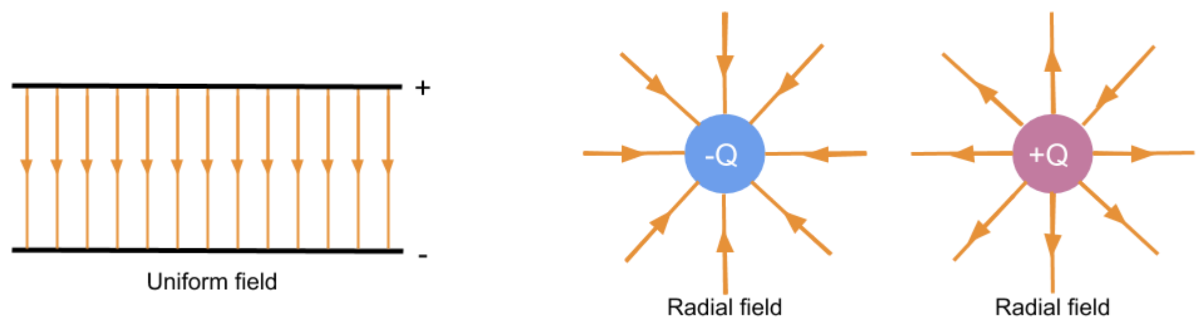
$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Where ε_0 is the permittivity of free space, **Q** is the charge, and **r** is the distance between the charges.

Electric potential difference (ΔV **)** is the energy needed to move a unit charge between two points.

4.4.99 - Field lines and equipotentials

Electric fields can be **uniform** or **radial** and can also be represented by the following field lines:



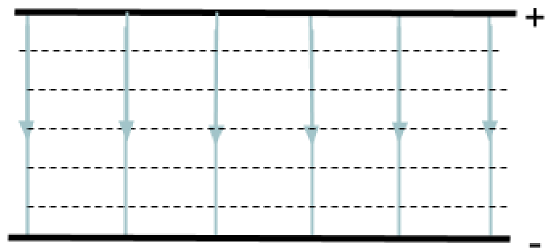
The field lines show the direction of the force acting on a **positive** charge. A **uniform field** exerts that **same** electric force everywhere in the field, as shown by the parallel and equally spaced field lines, whereas in a **radial field** the magnitude of electric force **depends on the distance** between the two charges.

The **distance** between field lines represents the magnitude of force, e.g. in the radial field diagrams above, as a charge moves further away from the centre, the magnitude of force would decrease because the distance between field lines increases.

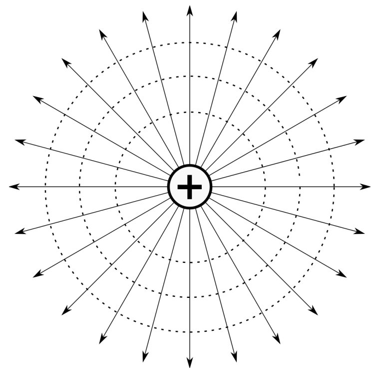
Electric fields have **equipotential surfaces**. The potential on an equipotential surface is the same everywhere, therefore when a charge moves along an equipotential surface, **no work is done**. Between two parallel plates the equipotential surfaces are planes which are equally spaced and parallel to the plates, whereas equipotential surfaces around a point charge form concentric circles.

You can draw equipotential surfaces by joining points of equal potential in an electric field together.

Equipotentials in a uniform field



Equipotentials in a radial field



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4.4.100 - Capacitance

Capacitance (C) is the charge stored by a capacitor per unit potential difference.

$$C = \frac{Q}{V}$$

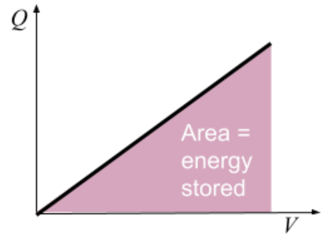
Where **Q** is the charge stored, and **V** is the potential difference across the capacitor.

4.4.101 - Energy stored by a capacitor

The electrical **energy stored by a capacitor (W)** is given by the **area under a graph of charge against potential difference**. As potential difference is directly proportional to charge, this graph forms a straight line through the origin, meaning the area underneath it is a right angle triangle so:

$$W = \frac{1}{2}\ QV$$

Where **Q** is the charge stored, and **V** is the potential difference across the capacitor.



You can derive two more variations of the above equation by substituting the formula for capacitance into it, as shown below:

$$W = \frac{1}{2}QV \qquad\qquad C = \frac{Q}{V}$$

For the first variation, rearrange the equation for capacitance so that its subject is charge (Q), and substitute it into the formula for the energy stored by a capacitor:

$$Q = CV$$

$$W = \tfrac{1}{2}(CV) \times V$$

$$W = \tfrac{1}{2}CV^2$$

For the second variation, rearrange the equation for capacitance so that its subject is potential difference (V), and substitute it into the formula for the energy stored by a capacitor:

$$V = \frac{Q}{C}$$

$$W = \tfrac{1}{2}Q \times \frac{Q}{C}$$

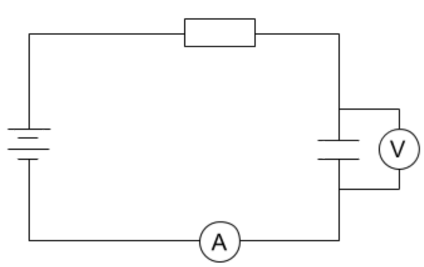
$$W = \frac{\frac{1}{2}Q^2}{C}$$

Therefore, you can find the energy stored by a capacitor using the following equations:

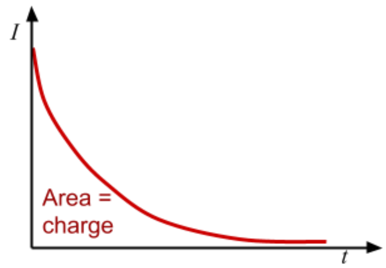
$$W = \frac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{\frac{1}{2}Q^2}{C}$$

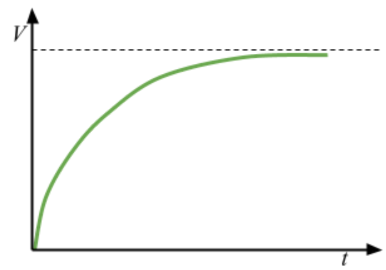
4.4.102 - Capacitor charging and discharging

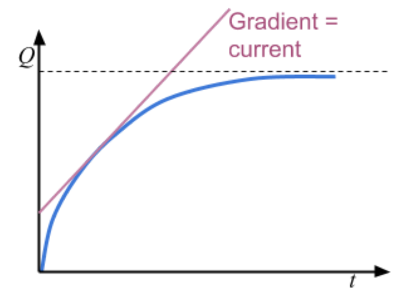
In order to charge a capacitor you must connect it in a circuit with a power supply and resistor, as shown in the circuit diagram below:



You could use a data logger to measure the values of potential difference and current in order to plot a graph of voltage and current against time. As $Q = I \times t$, you can draw a graph of **charge against time** by measuring the **area under the current-time graph**.

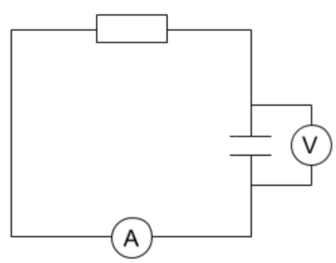




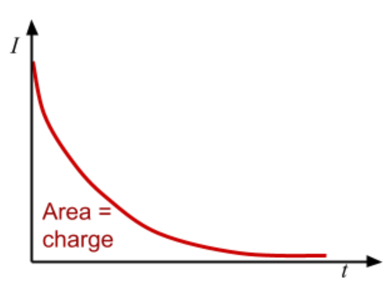


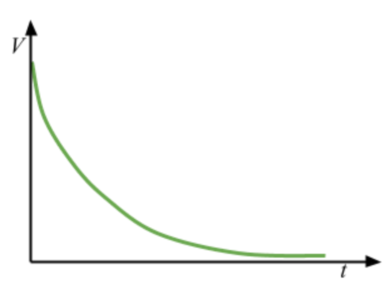
Once the capacitor is connected to a power supply, current starts to flow and negative charge builds up on the plate connected to the negative terminal. On the opposite plate, electrons are repelled by the negative charge building up on the initial plate, therefore these electrons move to the positive terminal and an **equal but opposite charge is formed on each plate**, creating a potential difference. As the **charge across the plates increases, the potential difference increases but the electron flow decreases** due to the force of **electrostatic repulsion** also increasing, therefore current decreases and eventually reaches zero.

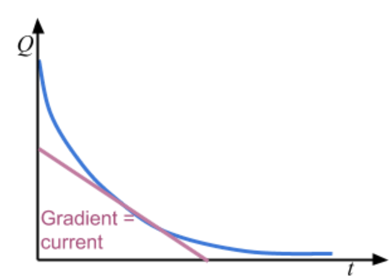
To discharge a capacitor through a resistor, you must connect it to a closed circuit with just a resistor.



Again you could use a data logger to measure voltage and current to plot graphs of **voltage, current and charge against time**.





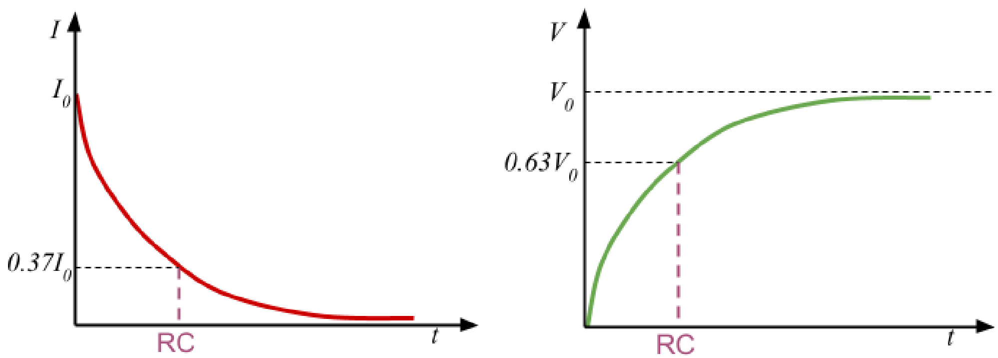


When the capacitor is discharging the **current flows in the opposite direction**, and the current, charge and potential difference across the capacitor will all fall exponentially, meaning it will take the same amount of time for each of the values to halve.

The **product of resistance and capacitance (RC)** is known as the **time constant**, and this is the value of time taken to:

- **Discharge a capacitor** to $\frac{1}{e} \approx 0.37$ **of its initial value** (of charge, current or voltage)
- **Charge a capacitor** to $(1 - \frac{1}{e}) \approx 0.63$ **of its initial value** (of charge or voltage)

You can **calculate the time constant from graphs of current, charge and voltage against time**, by finding the time where the values are either 0.37 of the initial value if discharging or 0.63 of the maximum value if charging (for charge or voltage), as shown in the graphs below.



4.4.103 - Equations for capacitor charging and discharging

The graph of charge against time follows an **exponential curve** for capacitor discharging, meaning that the equation to calculate the value of charge at a certain point in time involves an exponential function, as shown below:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

Where $\mathbf{Q_0}$ is initial charge, **t** is time, **C** is capacitance and **R** is resistance of the discharge circuit.

Using the equation above, you can derive related equations for calculating the current and potential difference of a discharging capacitor.

To derive the equation for calculating the potential difference, you must use the equation for **capacitance**:

$$Q = Q_0 e^{-\frac{t}{RC}} \qquad\qquad C = \frac{Q}{V}$$

Rearrange the capacitance equation so that its subject is charge (Q):

$$Q = CV$$

At t = 0, the charge is equal to $\mathbf{Q_0}$ and the voltage is equal to $\mathbf{V_0}$ (while capacitance is constant), so:

$$Q_0 = CV_0$$

Next, substitute the above equations into the equation for the charge of a discharging capacitor:

$$CV = CV_0 e^{-\frac{t}{RC}}$$

Finally, divide through by capacitance (C) to get:

$$V = V_0 e^{-\frac{t}{RC}}$$

To derive the equation for calculating the current, you must use the equation for potential difference above and the formula for **Ohm's law**:

$$V = V_0 e^{-\frac{t}{RC}} \qquad\qquad V = IR$$

At t = 0, the current is equal to $\mathbf{I_0}$ and the voltage is equal to $\mathbf{V_0}$ (while resistance is constant), so:

$$V_0 = I_0 R$$

Next, substitute the above (Ohm's law) equations into the equation for the voltage of a discharging capacitor:

$$IR = I_0 R e^{-\frac{t}{RC}}$$

Finally, divide through by capacitance (R) to get:

$$I = I_0 e^{-\frac{t}{RC}}$$

The corresponding **log equations** for charge, current and potential difference over time are:

$$ln\ Q = lnQ_0 - \frac{t}{RC} \qquad ln\ I = ln\ I_0 - \frac{t}{RC} \qquad ln\ V = ln\ V_0 - \frac{t}{RC}$$

Where $\mathbf{Q_0}$ is initial charge, $\mathbf{I_0}$ is initial current, $\mathbf{V_0}$ is initial voltage, **t** is time, **C** is capacitance and **R** is resistance of the discharge circuit.

You can derive all of these log equations in the **same** way, below is the derivation of the charge log equation.

Firstly, take the natural log of both sides of the discharging equation for charge:

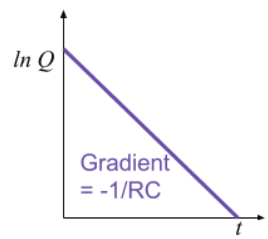
$$Q = Q_0 e^{-\frac{t}{RC}}$$
$$ln\ Q = ln(Q_0 e^{-\frac{t}{RC}})$$

Simplify the above equation using the rules **log(ab) = log(a) + log(b)**, **log($\mathbf{a^n}$) = nlog(a)** and **ln(e) = 1**.

$$\ln Q = \ln Q_0 + \ln e^{-\frac{t}{RC}}$$

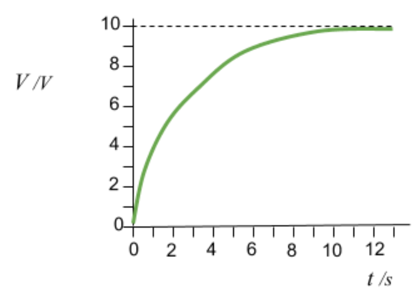
$$\ln Q = \ln Q_0 - \frac{t}{RC}$$

If you plot a graph of ln(Q) against t, the **gradient of this graph is** $\frac{-1}{RC}$, therefore $RC = \frac{-1}{gradient}$.

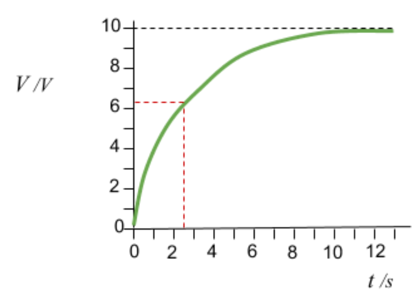


Below are some example questions using the above equations:

Find the approximate value of **time constant** for the capacitor charging graph below:



Firstly, draw a line across at **63%** of its maximum value as the time at which this occurs will be the time constant. Read off the value for time at this point.



The value of time constant is approximately **2.5 s**.

A capacitor with a capacitance of 300 μF is discharged through a 400 kΩ resistor, its initial value of charge is 5 C, find the value of charge after 5 s.

Use the formula $Q = Q_0 e^{-\frac{t}{RC}}$.

$$Q = 5 \times e^{-\frac{5}{300\times10^{-6}\times400\times10^{3}}} = 5 \times e^{\frac{-5}{120}} = \textbf{4.8 C}$$

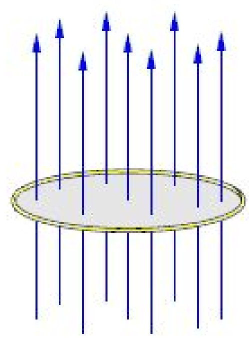
4.4.105 - Magnetic flux density, flux and flux linkage

The **magnetic flux density (B)** of a magnetic field, is a measure of the strength of the field, and it is measured in the unit Tesla.

Magnetic flux (ϕ) is a value which describes the magnetic field or magnetic field lines passing through a given area, and it is calculated by finding the product of magnetic flux density and the given area, **when the field is perpendicular to the area**:

$$\Phi = BA$$

Where **B** is the magnetic flux density, and **A** is the area.



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Magnetic flux linkage (Nϕ) is the **magnetic flux** multiplied by the **number of turns N**, of a coil:

$$N\Phi = BAN$$

Where **B** is the magnetic flux density, **A** is the area and **N** is the number of turns.

4.4.106 - Charged particles moving in a magnetic field

A force acts on charged particles moving in a magnetic field, this is why a force is exerted on a current-carrying wire in a magnetic field, because it contains moving electrons, which are negatively charged particles. The **magnitude of force (F)** exerted on a particle can be calculated using the following formula:

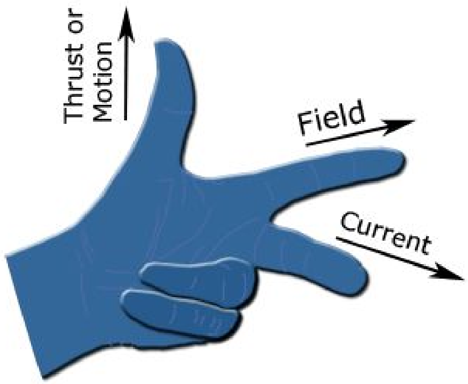
$$F = BQv \sin \theta$$

Where **B** is the magnetic flux density, **Q** is the charge, and **v** is the velocity of the particle.

In the equation above, **θ** is the angle between the **velocity** of the particle and the **direction** of the **magnetic field**.

To find the direction of the force exerted on a charged particle you can use **fleming's left hand rule**:
Spread out your thumb, first and second finger so that they are all **perpendicular** to each other, each finger represents a different quantity:

- **ThuMb** - represents the direction of the **Motion**/force
- **First finger** - represents the direction of the **Field**
- **SeCond finger** - represents the direction of the **Conventional Current** (opposite direction to electron flow)



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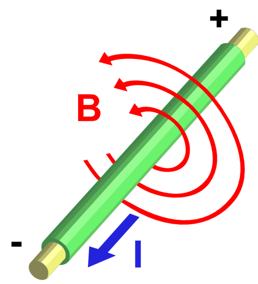
To use this rule simply point the respective fingers in the direction of two known values, for example conventional current and field, to find the direction of the third, in this case motion/force.

To find the direction of motion/force exerted on a charged particle you can use **Fleming's left hand rule**, using the **second finger as the direction of travel**, however if the **charge on the particle is negative, reverse the direction of your second finger**, because the se**C**ond finger represents **C**onventional **C**urrent, which flows from positive to negative.

The force exerted is always perpendicular to the motion of travel, which causes charged particles to follow a **circular path** when in a magnetic field, because the force induced by the magnetic field acts as a centripetal force.

4.4.107 - Current carrying conductors in a magnetic field

When current passes through a wire, a magnetic field is induced - this is true for any long, straight current-carrying conductor. The field lines of the induced magnetic field form **concentric rings** around the wire.



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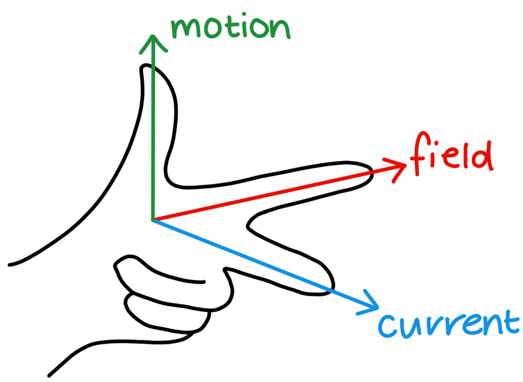
When a current-carrying wire is placed in a magnetic field, **a force is exerted on the wire**. To find the **magnitude of the force (F)** you can use the formula:

$$F = BIl \sin\theta$$

Where **B** is the magnetic flux density, **Q** is the charge, and **v** is the velocity of the particle.

In the equation above, **θ** is the angle between the **current** and the **direction of the magnetic field**.

To find the direction of motion/force exerted you can use **Fleming's left hand rule**, using the **second finger as the conventional current**.



4.4.108 - Induction of e.m.f in a coil through relative motion between the coil and a permanent magnet

When a conducting rod moves relative to a magnetic field, the electrons in the rod will experience a force (as they are charged particles), and build up on one side of the rod, causing an **emf to be induced** in the rod, this is known as **electromagnetic induction**. This phenomenon also occurs if you move a bar magnet relative to a coil of wire, **if the coil forms a complete circuit, a current is also induced**.

There are two laws which govern the effects of **electromagnetic induction**:

- **Faraday's law** - the magnitude of induced emf is equal to the rate of change of flux linkage
- **Lenz's law** - the direction of induced current is such as to oppose the motion causing it

Faraday's law can be expressed using the following equation:

$$\varepsilon = N \frac{\Delta\Phi}{\Delta t}$$

Where **ε** is magnitude of induced emf, and **N** $\frac{\Delta\Phi}{\Delta t}$ is rate of change of flux linkage.

Using the equation for **Faraday's law** (above), you can see that the factors affecting the emf induced in a coil when there is relative motion between the coil and a permanent magnet are:

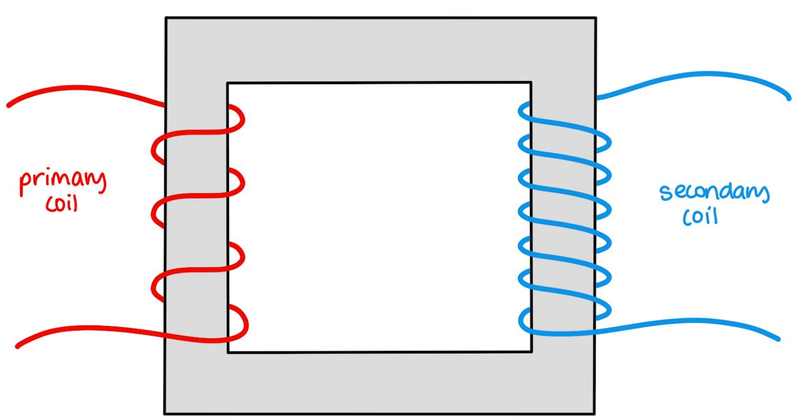
- The **number of turns in the coil (N)** -
  - This is directly proportional to the induced emf
- The **magnetic flux density (B)** of the field created by the permanent magnet -
  - This is directly proportional to the induced emf (as $\Phi = BA$)
- The **area of the cross section (A)** of the coil -
  - This is directly proportional to the induced emf (as $\Phi = BA$)
- The **time taken (t)** for the motion -
  - This is **inversely** proportional to the induced emf

4.4.109 - Induction of e.m.f in a coil through the change in current of another coil

A magnetic field is induced around a current-carrying wire, and so the same is true for a coil of current-carrying wire. If the current through this wire changes, the magnetic field will **also change**, meaning that an emf will be induced in a second coil if it is in the magnetic field - this is called **mutual inductance**. **The induced emf in the second coil is proportional to the change in current in the first coil**.

Note that if the second coil forms a complete circuit, a **current is also induced.**

An example of an application of mutual inductance is a transformer.



The factors affecting the emf induced in the second coil are:

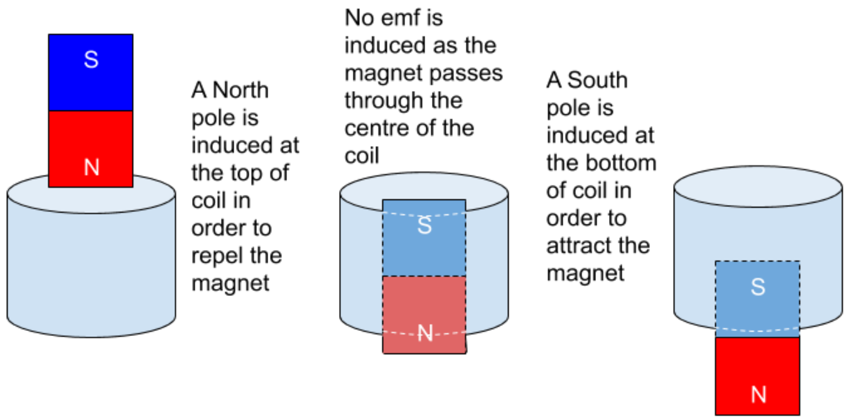
- The **magnetic flux density (B)** of the field created by the initial coil -
  - This is directly proportional to the induced emf
  - This is determined by the **number of turns (*l*)** and the **current (*I*)** flowing through the initial coil ($F = BIl \sin\theta$)
- The **distance between the two coils** -
  - This is **inversely** proportional to the induced emf
  - The **further** apart they are, the less magnetic flux passes through the second coil and so the induced emf is **lower**
- The **number of turns (N)** in the second coil -
  - This is directly proportional to the induced emf
- The **area of the cross section (A)** of the second coil -
  - This is directly proportional to the induced emf
- The **time taken (t)** for change in current -
  - This is **inversely** proportional to the induced emf

4.4.110 - Faraday's law and Lenz's law

Lenz's law states that the direction of induced current is such as to oppose the motion causing it, and you can use it to predict the direction of an induced emf.

To demonstrate **Lenz's law**, you can measure the speed of a magnet falling through a coil of wire, and its speed when falling from the same height without falling through the coil. What you will find is that the magnet takes longer to reach the ground when it moves through the coil, this can be explained by Lenz's law:

1. As the magnet approaches the coil, there is a **change of flux through the coil so an emf and a current is induced.**
2. Due to Lenz's law, **the direction of induced current is such as to oppose the motion of the magnet** so the **same pole** as the pole of the magnet approaching the coil will be induced, in order to repel the magnet. This causes the magnet to slow down, due to electrostatic forces of **repulsion**.
3. As the magnet passes through the centre of the coil, there is no change in flux so no emf is induced.
4. As the magnet begins to leave the coil, **there is a change in flux, so a current is induced that opposes the motion** of the magnet. Therefore, an **opposite pole** is induced by the magnet causing it to slow down once again, due to electrostatic forces of **attraction**.



Lenz's law is a direct consequence of the **conservation of energy** - it ensures that the electrical energy gained by the induction of a current is offset by an equal amount of energy being removed.

To exemplify this, consider if the opposite to Lenz's law was true - the direction of induced current is such as to support the motion causing it, and consider the falling magnet example above.

1. As the magnet approaches the coil, there is a change of flux through the coil so an emf and a current is induced.
2. Due to the opposite of Lenz's law (which we are assuming to be true), **the direction of the induced current supports the motion of the magnet** so the **opposite pole** to the one of the magnet which is approaching the coil is induced, in order to attract the magnet.
3. This causes the magnet to speed up, due to electrostatic forces of **attraction**, so the magnet gains kinetic energy.

The energy of the system has increased through the induction of the current and also through the acceleration of the magnet - energy has been **created** from nothing! This is a **violation** of the **conservation of energy**, which is why Lenz's law is as it is.

You can use **Faraday's law** to determine the magnitude of induced emf (ε) by using the following equation, which describes Faraday's law:

$$\varepsilon = N \frac{\Delta\Phi}{\Delta t}$$

Where **N** is the number of turns, **ϕ** is the magnetic flux, and **t** is the time.

If you combine Faraday's law and Lenz's law you get the following equation, which shows that the magnitude of induced emf is directly proportional to the rate of change of flux linkage, and is in such a direction as to induce a current which opposes the motion causing the induction of emf.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

The above equation could be rewritten using the **derivative** of magnetic flux linkage with respect to time:

$$\varepsilon = \frac{-d(N\Phi)}{dt}$$